γZ exchange corrections in parity-violating electron scattering

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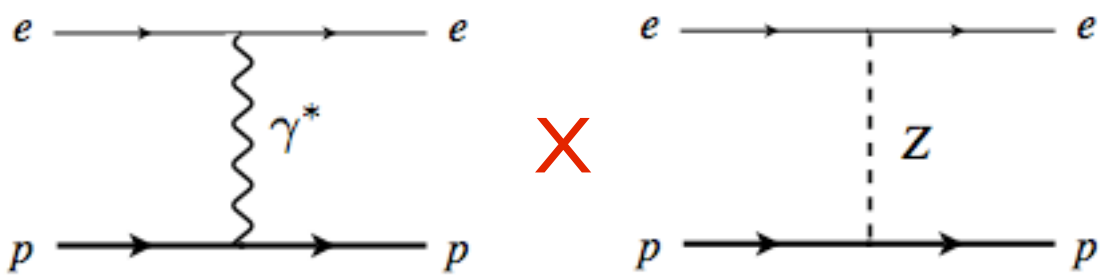
Parity-violating electron scattering

right-left polarization asymmetry in $\vec{e} + p \to e + p$ scattering

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = \frac{2\Re\{\mathcal{M}_\gamma^\dagger \mathcal{M}_Z\}}{|\mathcal{M}_\gamma|^2}$$



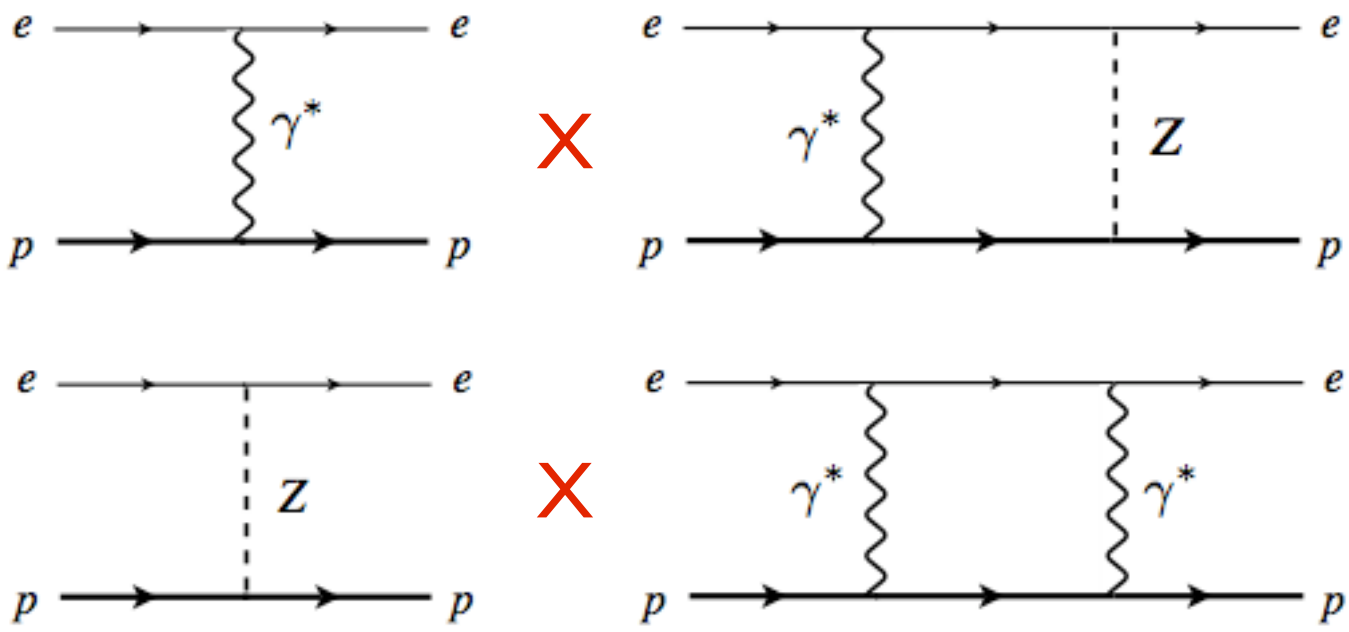
$$j_\mu^Z = \bar{u}\left(g_V^e \gamma_\mu + g_A^e \gamma_\mu \gamma_5\right) u$$

$$J_Z^\mu(q) = \bar{U}\left(F_1^Z \gamma^\mu + F_2^Z i \frac{\sigma^{\mu\nu} q_\nu}{2M} + G_A^Z \gamma^\mu \gamma_5\right) U$$

$$g_V^e = -(1 - 4s_W^2)$$
$$g_A^e = +1$$
$$F_i^Z = (1 - 4s_W^2) F_i^p - F_i^n - F_i^s$$
$$G_A^Z = -G_A \tau_3 + G_A^s$$

using relation between weak and EM form factors

Two-boson exchange corrections



"$\gamma(Z\gamma)$"

"$Z(\gamma\gamma)$"

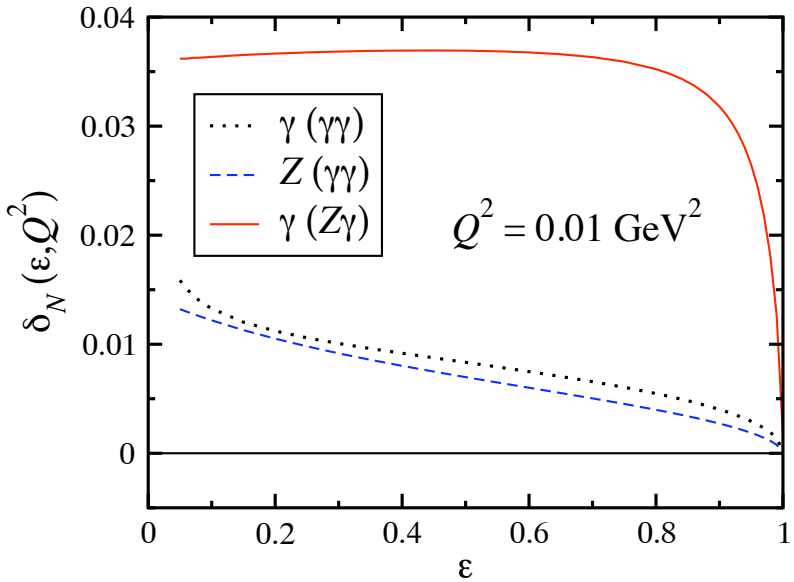
Electromagnetic radiative corrections interfere with M_Z ($M_\gamma \rightarrow M_\gamma + M_{\gamma\gamma}$)

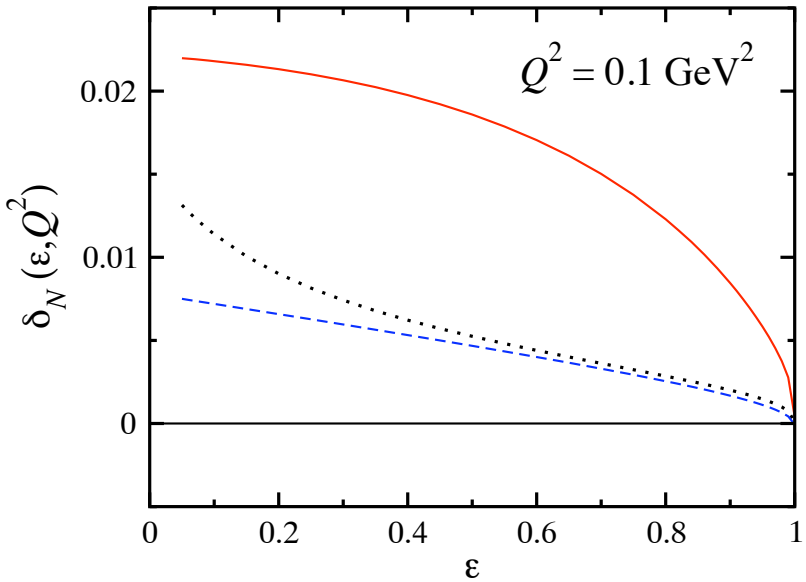
plus weak radiative corrections interfere with M_γ ($M_Z \rightarrow M_Z + M_{\gamma Z}$)

plus two-photon exchange "$\gamma(\gamma\gamma)$" in denominator

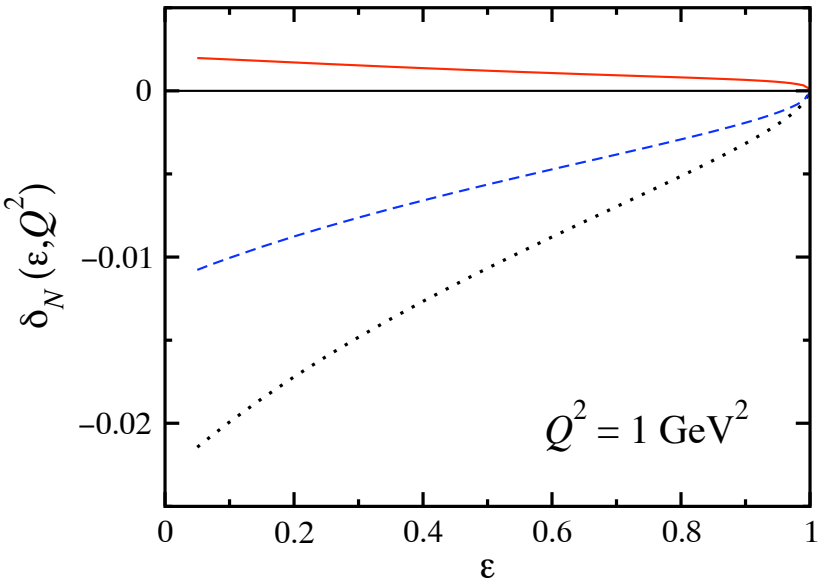
Zhou, Kao & Yang, PRL 2007; Tjon & Melnitchouk, PRL 2008;
Tjon, Melnitchouk & Blunden, PRC 2009

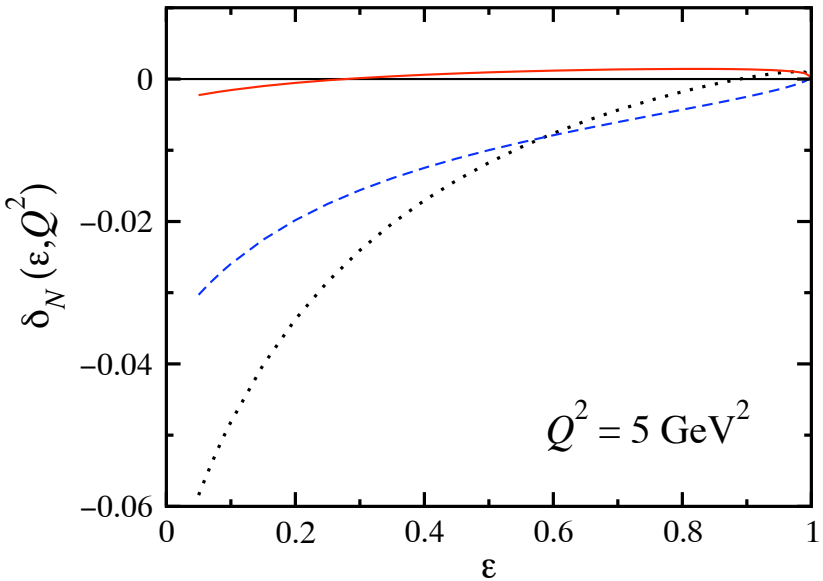
Tjon, Blunden & Melnitchouk, PRC (2009)





$$\delta \approx \delta_{Z(\gamma\gamma)} + \delta_{\gamma(Z\gamma)} - \delta_{\gamma(\gamma\gamma)}$$





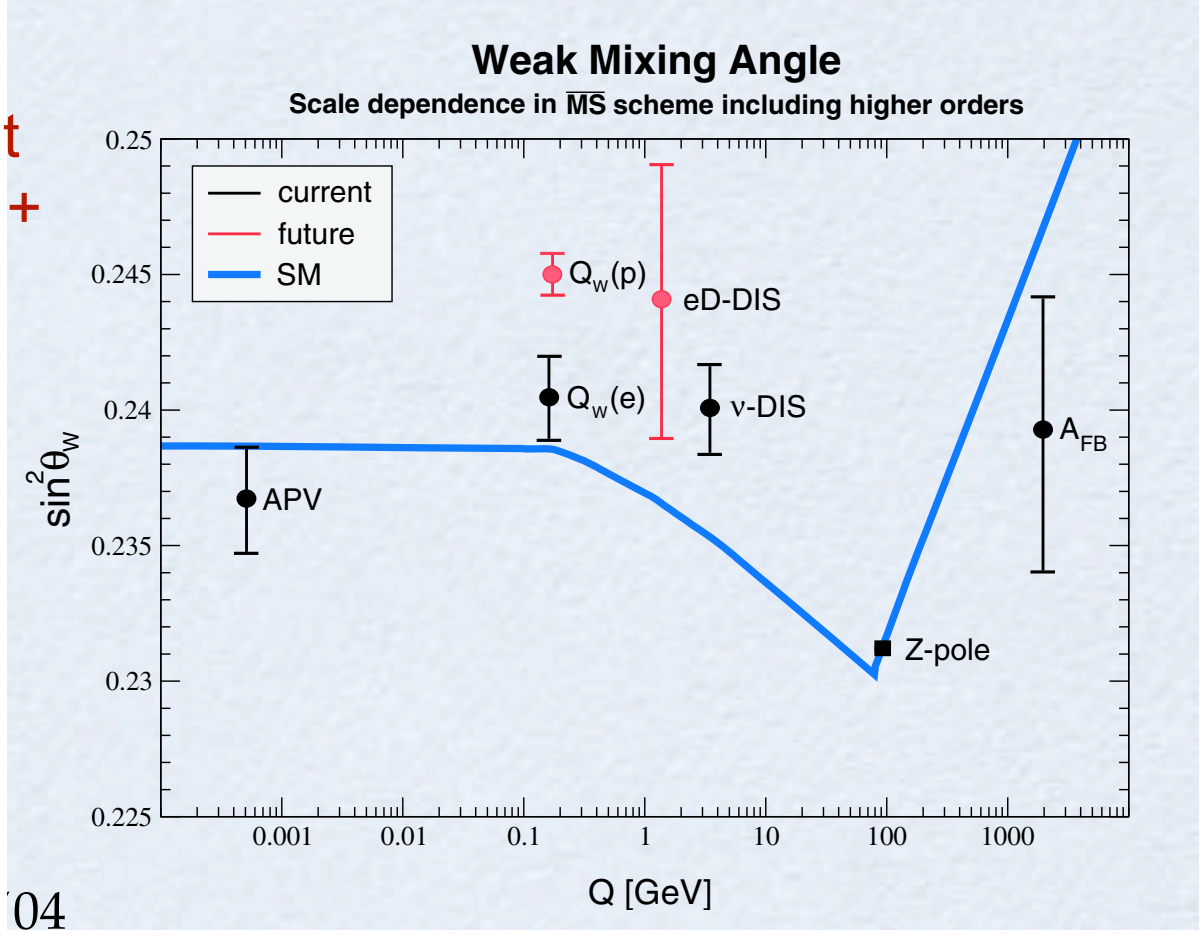
Weak charge of the proton: $Q_W^p = 1 - 4\sin^2\theta_W \approx 0.072$

$$\sin^2\theta_W(M_Z^2) = 0.23113 \pm 0.00015$$ (PDG)

$$\sin^2\theta_W(0) = 0.23807 \pm 0.00017$$ (Erler et al., 2004)

Qweak: 4% measurement of weak charge (2% expt + 2% theory)

0.3% measurement of weak mixing angle



'04

$$A^{\mathrm{PV}} \equiv \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \rightarrow \frac{G_F}{4\pi\alpha\sqrt{2}}\, t\, Q_W^p$$

Measure at forward angles where $t \rightarrow 0$

- At tree level proton's weak charge given by

$$Q_{\mathrm{w}}^p = 1 - 4\sin^2\theta_W$$

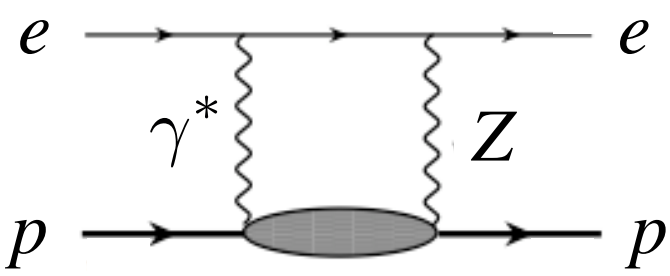
- At higher orders Q_{w}^p receives corrections from electroweak quantum fluctuations

$$\begin{aligned} Q_W^p &= (1 + \Delta\rho + \Delta_e)(1 - 4\sin^2\theta_W(0) + \Delta_e') \\ &\quad + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z}(0) \end{aligned}$$

Allow for E-dependence

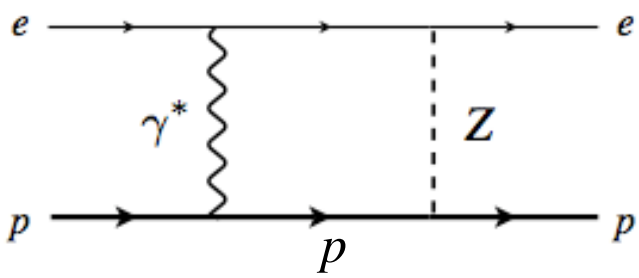
- QED and short-distance corrections under control

  $\longrightarrow$ most uncertain is γZ box contribution, sensitive to *long-distance* physics



- First estimates of γZ boxes by Marciano & Sirlin (MS) in atomic parity violation (APV)

  $\longrightarrow$ low-energy part approximated by Born contribution (elastic intermediate state)



  $\longrightarrow$ high-energy part (above scale $\Lambda \sim 1\,\mathrm{GeV}$) computed in terms of scattering from free quarks

Marciano, Sirlin, PRD ***29****, 75 (1984)*

Two parity-violating contributions

$$\Box_{\gamma Z}(E) = \Box^{A}_{\gamma Z}(E) + \Box^{V}_{\gamma Z}(E)$$

$\boxed{V_e \times A_h}$ $\longrightarrow$ computed by MS $\quad \Box^{A}_{\gamma Z}(E)$

$$\delta^{(\gamma Z)} Q^p_{\rm w} = \frac{5\alpha}{2\pi}(1 - 4\sin^2\theta_W)\left[\ln\frac{M_Z^2}{\Lambda^2} + C_{\gamma Z}(\Lambda)\right]$$

high-energy $\quad$ low-energy

$\longrightarrow$ estimated uncertainty $\Delta Q^p_{\rm w} = 0.65\%$

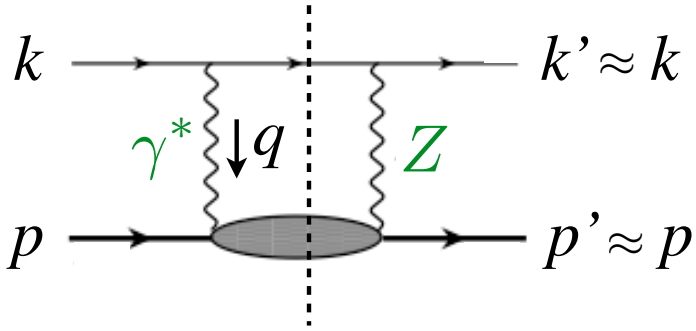
Erler et al., PRD ***68****, 016006 (2003)*

$\boxed{A_e \times V_h}$ $\longrightarrow$ small at low E_e $\quad \Box^{V}_{\gamma Z}(E)$

$\longrightarrow$ neglected in APV, but is it small at GeV energies?

- A_e x V_h term recently computed in forward limit within dispersion relation (DR) approach

Gorchtein, Horowitz, PRL ***102****, 091806 (2009)*



forward limit

$$t = (k - k')^2 \to 0$$

$$s = (k + p)^2 = M(M + 2E)$$

$$\Re e\, \Box^V_{\gamma Z}(E) = \frac{1}{\pi} P \int_{-\infty}^{\infty} dE' \frac{\Im m\, \Box^V_{\gamma Z}(E')}{E' - E}$$

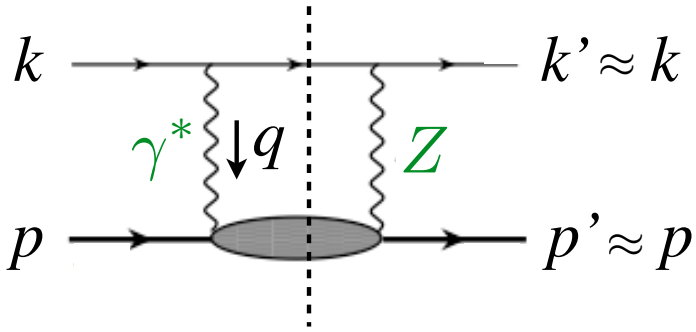
Negative energy corresponds to crossed box

By optical theorem

$$2\Im m\, \mathcal{M}_{fi} = \int d\rho \sum_n \mathcal{M}^*_{nf} \mathcal{M}_{ni}.$$

- A_e x V_h term recently computed in forward limit within dispersion relation (DR) approach

Gorchtein, Horowitz, PRL ***102****, 091806 (2009)*



forward limit

$$t = (k - k')^2 \to 0$$

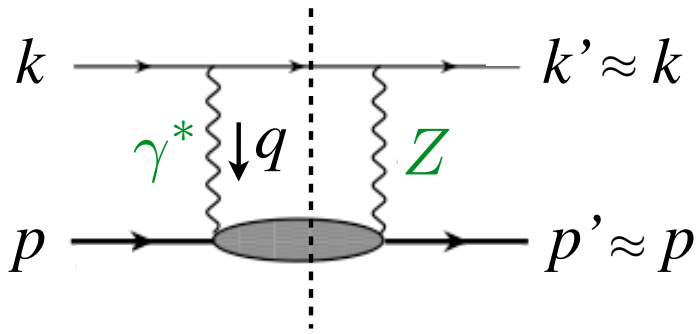
$$s = (k + p)^2 = M(M + 2E)$$

$$2\,\Im m\,\mathcal{M}^{\rm PV}_{\gamma Z} = 4\pi M \int \frac{d^3k'}{(2\pi)^3 2E_{k'}} \left(\frac{4\pi\alpha}{Q^2}\right)\left(\frac{-2G_F}{\sqrt{2}}\right) \times \frac{1}{1+Q^2/M_Z^2} L^{\gamma Z}_{\mu\nu} W^{\mu\nu}_{\gamma Z}$$

$$W^{\mu\nu}_{\gamma Z} = \frac{1}{4\pi M}\int d^4x\, e^{iq\cdot x} \times \langle p \,|\, [J^\mu_\gamma(x) J^\nu_Z(0) + J^\mu_Z(x) J^\nu_\gamma(0)] \,|\, p\rangle = \frac{1}{M}\left[-F_1^{\gamma Z} g^{\mu\nu} + F_2^{\gamma Z}\frac{p^\mu p^\nu}{p\cdot q}\right]$$

- A_e x V_h term recently computed in forward limit within dispersion relation (DR) approach

Gorchtein, Horowitz, PRL ***102****, 091806 (2009)*



forward limit

$t = (k - k')^2 \to 0$

$s = (k + p)^2$

$= M(M + 2E)$

$$\Im m\, \Box^V_{\gamma Z}(E) = \frac{\alpha}{(s - M^2)^2} \int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{\max}} \frac{dQ^2}{1 + Q^2/M_Z^2}$$

$$\times \left[F_1^{\gamma Z} + F_2^{\gamma Z} \frac{s\left(Q^2_{\max} - Q^2\right)}{Q^2\left(W^2 - M^2 + Q^2\right)} \right]$$

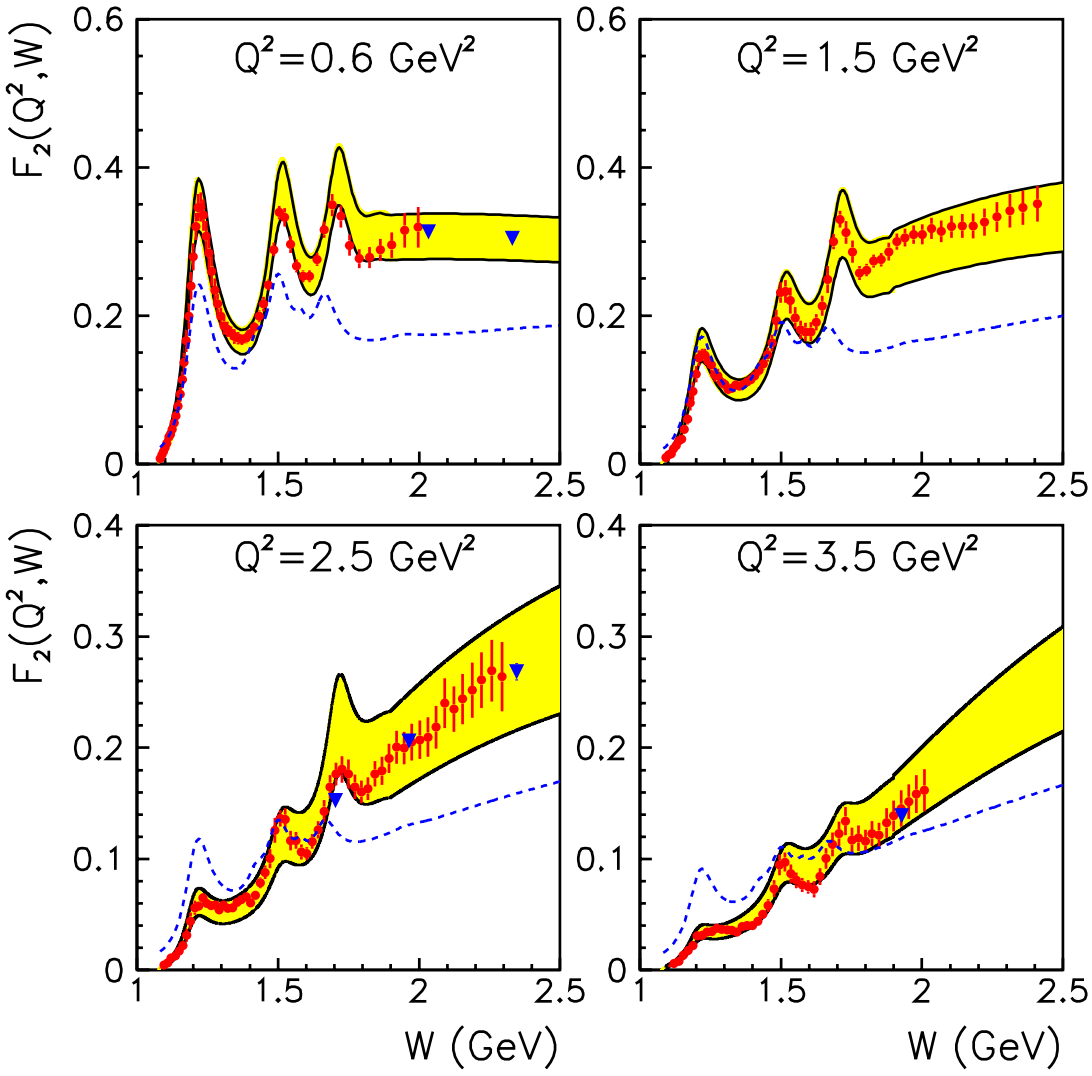
Sibirtsev, Blunden, Melnitchouk & Thomas (submitted to PRD)

Resonance region (W<2.5 GeV)

- For isospin 3/2 states, CVC and isospin symmetry imply

$$F^{\gamma Z} = (1 + Q_W^p)\, F^{\gamma}$$

- For isospin 1/2 states, transition couplings with few percent
- Use phenomenological input
- Take P33 (1232), D13 (1520), F15(1680), F37 (1950) plus background



Deep inelastic region

- Approximate interference structure functions

$$F_{1,2}^{\gamma Z} \approx F_{1,2}^{\gamma} \equiv F_{1,2}$$

$$F_2^{\gamma Z} = x \sum_q 2 e_q g_V^q \, (q + \bar{q})$$

$$F_2^{\gamma} = x \sum_q e_q^2 \, (q + \bar{q})$$

$\longrightarrow$ good approximation at low x

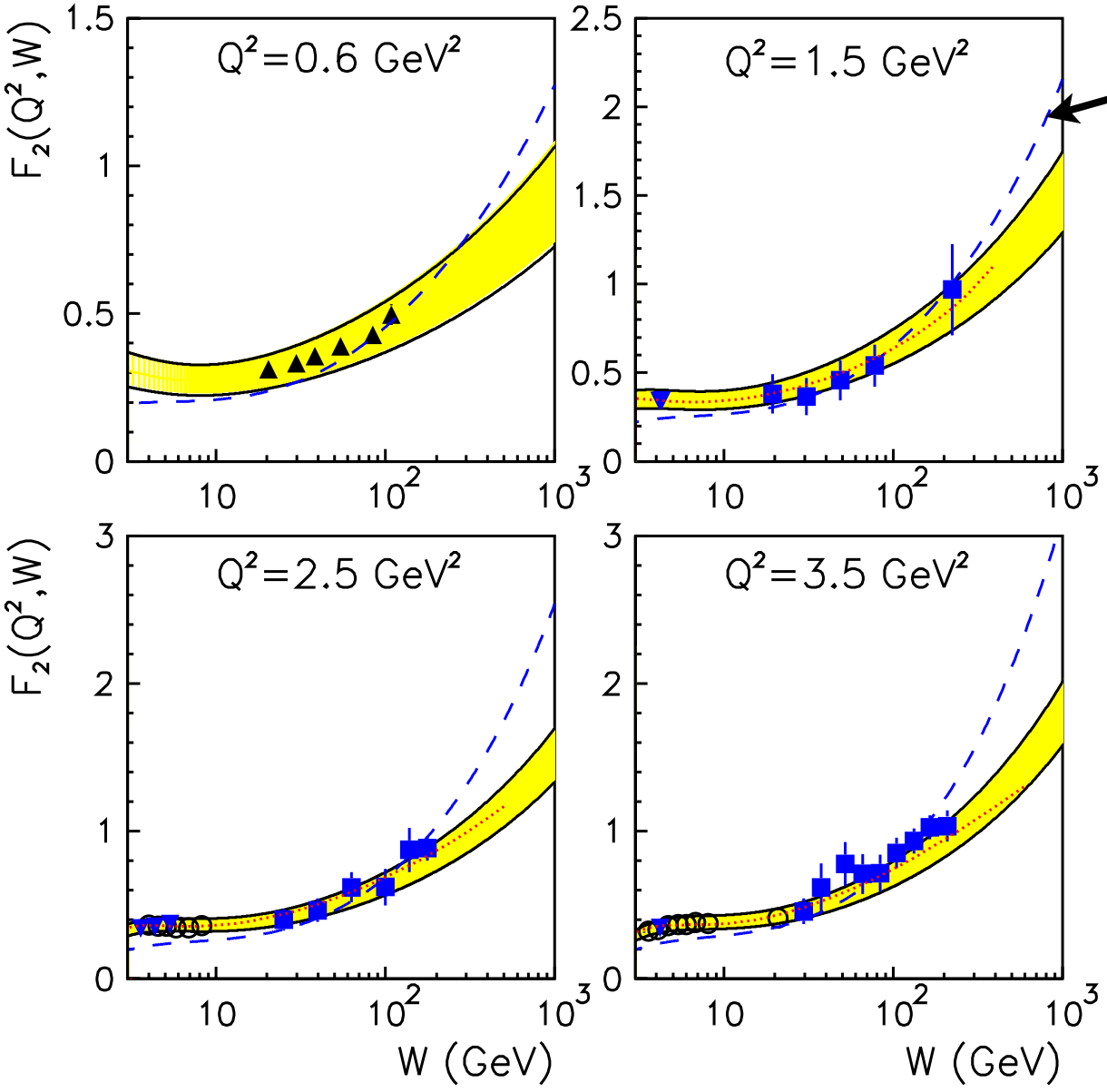
$\longrightarrow$ provides upper limit at larger x

• F_2 parametrization motivated by Regge theory; valid at both low and high Q^2

• Pomeron (related to sea quark), and Reggeon (related to valence quark) components

$$F_2(x, Q^2) = A_P \, x^{-\Delta} (1-x)^{n+4} \left[\frac{Q^2}{Q^2 + \Lambda_P^2} \right]^{1+\Delta} + A_R \, x^{1-\alpha_R} (1-x)^n \left[\frac{Q^2}{Q^2 + \Lambda_R^2} \right]^{\alpha_R}$$

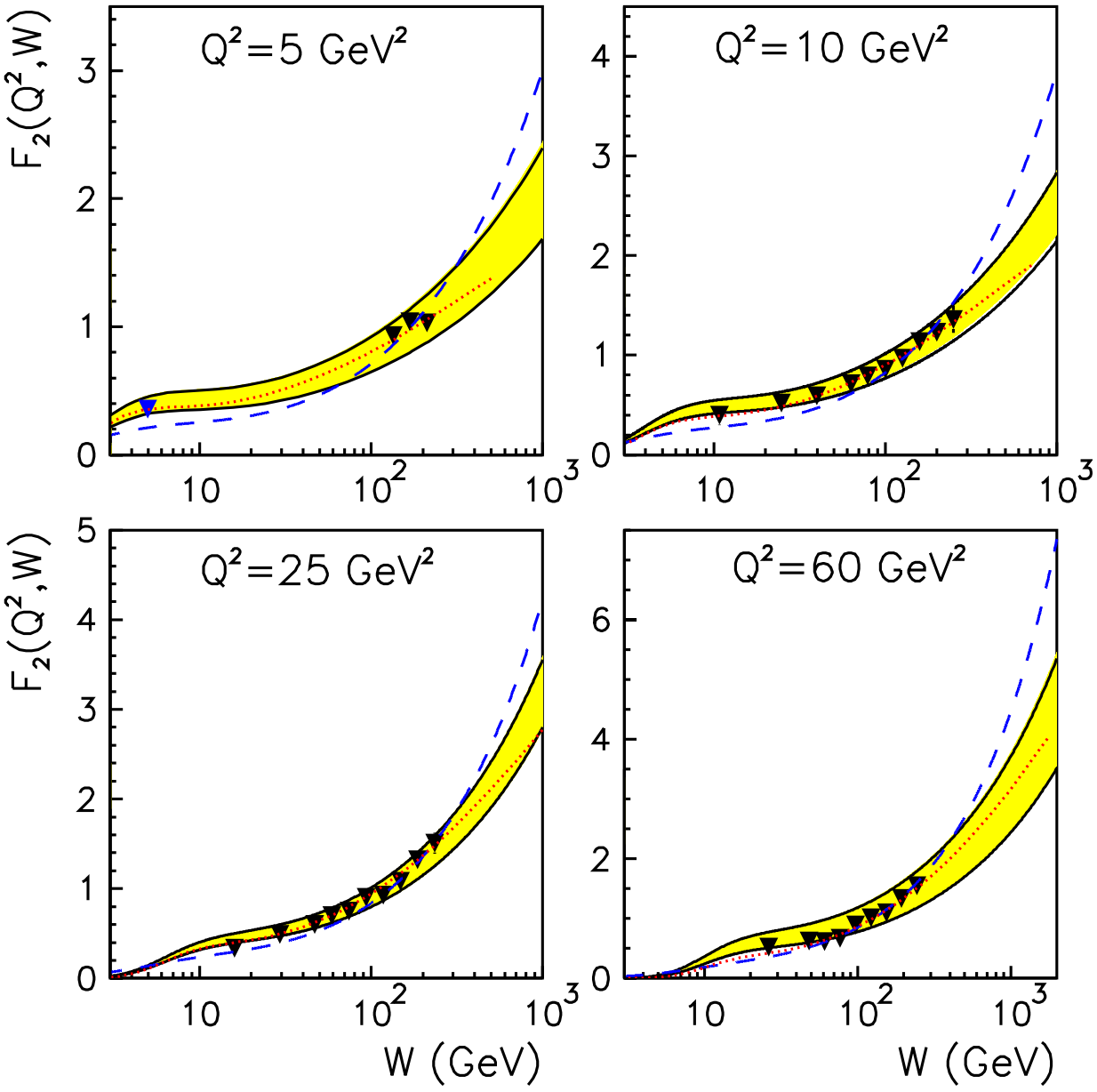
Large W, low Q^2



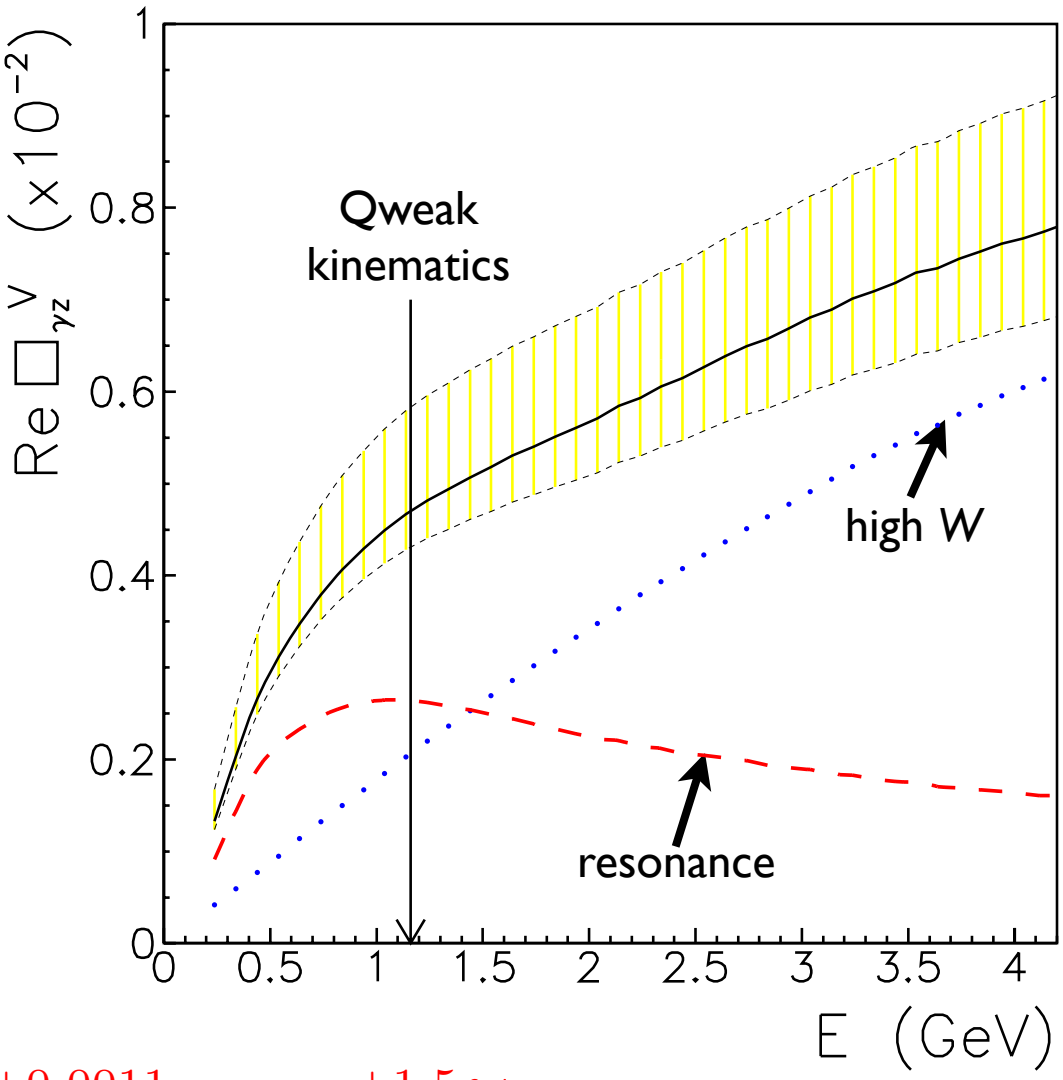
Cvetic et al, EPJC ***20****, 77 (2001)*

used as input in GH model

Large W, large Q^2



Performing dispersion integral: real part of correction



$0.0047^{+0.0011}_{-0.0004}$ or $6.6^{+1.5}_{-0.5}\%$

Summary

- Qweak correction large, but uncertainty under control
- Uncertainty in Qweak may be reduced further with measurements of γZ interference structure functions in PVDIS
- 50% larger than GH result
- Dispersion relations that use cross section data are useful at forward angles, however still need for models to extrapolate (not all data is available, e.g. γZ interference, axial part)

Collaborators: Sibirtsev, Melnitchouk, Thomas